ELEMENTARY AND MIDDLE SCHOOL MATHEMATICS

Teaching Developmentally

EIGHTH EDITION

JOHN A. VAN DE WALLE
KAREN S. KARP JENNIFER M. BAY-WILLIAMS

Elementary and Middle School Mathematics

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Teaching Developmentally

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About the Authors

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Jennifer M. Bay-Williams is a professor of mathematics education at the University of Louisville (Kentucky). Jennifer has published many articles on teaching and learning in NCTM journals. She has also co-authored numerous books, including *Developing Essential Understanding of Addition and Subtraction for Teaching Mathematics in Pre-K–Grade 2*, *Math and Literature: Grades 6–8*, *Math and Nonfiction: Grades 6–8*, and *Navigating Through Connections in Grades 6–8*. She is the author of the *Field Experience Guide* for this book. Jennifer taught elementary, middle, and high school in Missouri and in Peru, and continues to work in classrooms at all levels with students and with teachers. Jennifer is on the board of directors for TODOS: Equity for All, is the editor for the 2012 NCTM Yearbook, and is a former president of the Association of Mathematics Teacher Educators (AMTE).

About the Contributor

Jonathan Wray is the technology contributor to *Elementary and Middle School Mathematic: Teaching Developmentally* (Sixth–Eighth Editions). He is the instructional facilitator for secondary mathematics curricular programs in the Howard County public school system. He is the president elect of the Association of Maryland Mathematics Teacher Educators (AMMTE) and past president of the Maryland Council of Teachers of Mathematics (MCTM). He has been recognized for his expertise in infusing technology in mathematics teaching, receiving the Outstanding Technology Leader in Education award for his school district from the Maryland Society for Educational Technology (MSET). Jon is also actively engaged in NCTM, serving on the editorial panels of *Teaching Children Mathematics* and *ON-Math*. He has served as a primary and intermediate grades classroom teacher, gifted/talented resource teacher, elementary mathematics specialist, curriculum and assessment developer, grant project manager, and educational consultant.

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Detailed Contents

SECTION I

Teaching Mathematics: Foundations and Perspectives

The fundamental core of effective teaching of mathematics combines an understanding of how children learn, how to promote that learning by teaching through problem solving, and how to plan for and assess that learning on a daily basis. Introductory chapters in this section provide perspectives on trends in mathematics education and the process of doing mathematics. These chapters develop the core ideas of learning, teaching, planning, and assessment. Additional perspectives on mathematics for children with diverse backgrounds and the role of technology are also discussed.

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SECTION II
Development of Mathematical Concepts and Procedures

This section serves as the application of the core ideas of Section I. Here you will find chapters on every major content area in the pre-K–8 mathematics curriculum. Numerous problem-based activities to engage students are interwoven with a discussion of the mathematical content and how children develop their understanding of that content. At the outset of each chapter, you will find a listing of "Big Ideas," the mathematical umbrella for the chapter. Also included are ideas for incorporating children's literature, technology, and assessment. These chapters are designed to help you develop pedagogical strategies and to serve as a resource for your teaching now and in the future.

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Preface

NEW TO THIS EDITION

The eighth edition has been revised to include the following changes to better prepare teachers to teach mathematics to all learners:

- **New adaptations and accommodations for English language learners and students with disabilities** appear not only in the narrative in Section I but also in many activities through direct examples and descriptions for the various content areas in Section II. The increased emphasis on diversity will be obvious to those who have used the book in the past. Chapter 4 (Planning in the Problem-Based Classroom) has an increased focus on planning for all learners, including new coverage on considerations for students with disabilities to complement the revised section on ELLs. Chapter 6 (Teaching Mathematics Equitably to All Children) contains significant updates to each section and alignment with the research synthesis from *RtI Practice Guide for Students Struggling in Mathematics* (Gersten, Beckmann, Clarke, Foegen, Marsh, Star, & Witzel, 2009). New to this chapter are strategies for cultural/ethnic differences, including Table 6.4, "Reflective Questions to Focus on Culturally Responsive Mathematics Instruction," and additional guidance for teachers on students with disabilities, including a chart of common stumbling blocks. Importantly, in Section II the Activities feature specific adaptations and accommodations for ELLs and students with disabilities. These provide specific ways to make the activity accessible and still challenging.
- **Revised Expanded Lessons** located in the book, the *Field Experience Guide*, and on MyEducationLab (www.myeducationlab.com) now include tips and strategies for English language learners and students with disabilities.
- **Increased emphasis on student misconceptions and how to address them effectively** will better support teachers' understanding of what needs explicit attention when teaching mathematics. Since the publication of the seventh edition, an increasing body of research has emerged on students' misconceptions and naïve understandings in a variety of mathematics content. Throughout Section II, the research about misconceptions and gaps in student mathematical knowledge is presented to assist teachers in the identification and preparation for these common barriers to understanding. In such topics as fractions and decimals, related findings allow teachers to plan ahead using examples and counterexamples to strengthen student understanding as they face what may be expected areas of confusion.
- **New samples of authentic student work** illustrate student thinking. Responses to problem-based assignments present glimpses into how students think about problems and what students' written work on mathematical tasks looks like, increasing teachers' awareness of how rich students' mathematical thinking can be—and how high our expectations should be. Some student work also demonstrates naïve understandings.
- **Increased early childhood coverage** provides expanded emphasis on and reorganization of early numeracy in Chapters 8 and 9 reflecting the work of the Committee on Early Childhood Mathematics through the National Research Council. Based on learning trajectories and progressions for the core areas of number, relations, and operations, the work with early learners is seen as the essential foundation for number sense and problem solving.

- **New Formative Assessment Notes** in each chapter in Section II guide readers through ideas they can test with individual students or students in groups. Formative assessment is one of the key tools in finding out what students are thinking, and thereby identifying their areas of strength and weakness. Chapter 5, Building Assessment into Instruction, contains a more detailed description of formative assessments organized by Piaget's three major assessment areas: tasks, observation, and interviews. To bring these ideas to life and to make them more directly linked to the content, these Formative Assessment Notes are included throughout content area chapters to support teachers in the effective use of formative assessment, which is directly connected to increased student achievement.
- **New information on using NCTM standards and *Common Core State Standards*** to inform instruction appears in Chapter 1 and 2 and in relevant references in Section II. Not surprisingly, this book is aligned with the new *Common Core State Standards*, adopted by 44 of the 50 states at the time of publication. The *Common Core State Standards* and other standards documents are described in Chapter 1. The Standards for Mathematical Practice portion of the *Common Core State Standards* are addressed in Chapter 2 (connected to the *Adding it Up* mathematical proficiencies) and infused throughout the book. As essential content is described in Section II, specific standards are referenced, giving the appropriate grade level and treatment relevant to the content. In addition, chapter content has been adapted to reflect the attention given to the content in the *Common Core State Standards*. Appendix A provides the Standards for Mathematical Practice.
- **Extensively updated information on how to effectively integrate new technological tools** to support teaching and learning appears in Chapter 7 and throughout the text with marginal icons. Updated technology integration content and strategies now also appear in select Activities.
- **A reorganization of Chapters 12 and 13** emphasizes both strategies for computation and estimation for addition and subtraction in Chapter 12 and the same for multiplication and division in Chapter 13. This is a change from the seventh edition, which separated developing strategies for whole number computation and estimation for the four operations. Many reviewers suggested this change, infusing computational estimation in these new chapters, and this rearrangement links too to the developmental nature of those operations.
- **A discussion on engaging families in meaningful ways** to help students learn mathematics appears in Chapter 4.
- **Additional attention to classroom discourse** now appears in Chapter 3 (Teaching Through Problem Solving). The coverage includes how to conduct productive discussion sessions and develop effective questioning, and is illustrated with a vignette.

OTHER CHANGES OF NOTE

Much has changed on the landscape of mathematics education, and so many aspects of the book have been updated to reflect those changes. In addition to the changes listed previously, the following substantive changes have been made:

- A new section on homework and parental involvement is provided in Chapter 4.
- There is an increased focus on the research-based three-phase developmental model of developing basic facts, and added new activities to support basic fact mastery appear in Chapter 10.
- The content on algebraic thinking has been adapted to align with current research and standards. Specifically, Kaput's five areas (from his 1999 work) are now three areas (based

on his 2008 work). Also, there is an increased emphasis on equivalence and variables, including adding the number-line representation of variables. Increased attention is given to making the properties (especially distributive) more explicit in response to the *Common Core State Standards*.

- Chapter 15 (Developing Fraction Concepts) has greatly expanded sections on partitioning and on equivalence to reflect three recent research reviews that have indicated that this is essential to all advanced fraction work and success in algebra.
- Chapter 16 (Developing Strategies for Fraction Computation) now includes Activities—ten new ideas for developing understanding of fraction operations.
- Chapter 18 has been shortened, had new activities added, and been refocused to address understanding of ratios more deeply (with less focus on connecting to other content areas).
- The chapter on measurement, Chapter 19, has been reorganized. Previously the development of all measurement formulas was shared at the end of the chapter; now the formulas are integrated with the corresponding measurement topic (e.g., area or volume).
- Chapter 21 gives more explicit attention to distinguishing between numerical data and categorical data.
- Chapter 23 includes a significantly revised section on order of operations and numerous new activities.

WHAT YOU WILL FIND IN THIS BOOK

If you look at the table of contents, you will see that the chapters are separated into two distinct sections. The first section, consisting of seven chapters, deals with important ideas that cross the boundaries of specific areas of content. The second section, consisting of 16 chapters, offers teaching suggestions and activities for every major mathematics topic in the pre-K–8 curriculum. Chapters in Section I offer perspectives on the challenging task of helping students learn mathematics. Having a feel for the discipline of mathematics—that is, to know what it means to "do mathematics"—is critical to learning how to teach mathematics well. In addition, understanding constructivist and sociocultural perspectives on learning mathematics and how that is applied to teaching through problem solving provides a foundation and rationale for how to teach and assess pre-K–8 students.

Importantly, you will be teaching diverse students, including students who are English language learners, are gifted, or have disabilities. You will learn how to apply instructional strategies in ways that support and challenge *all* learners. Formative assessment strategies, strategies for diverse learners, and effective use of technological tools are addressed in specific chapters in Section I (Chapters 5, 6, and 7, respectively), and throughout Section II chapters.

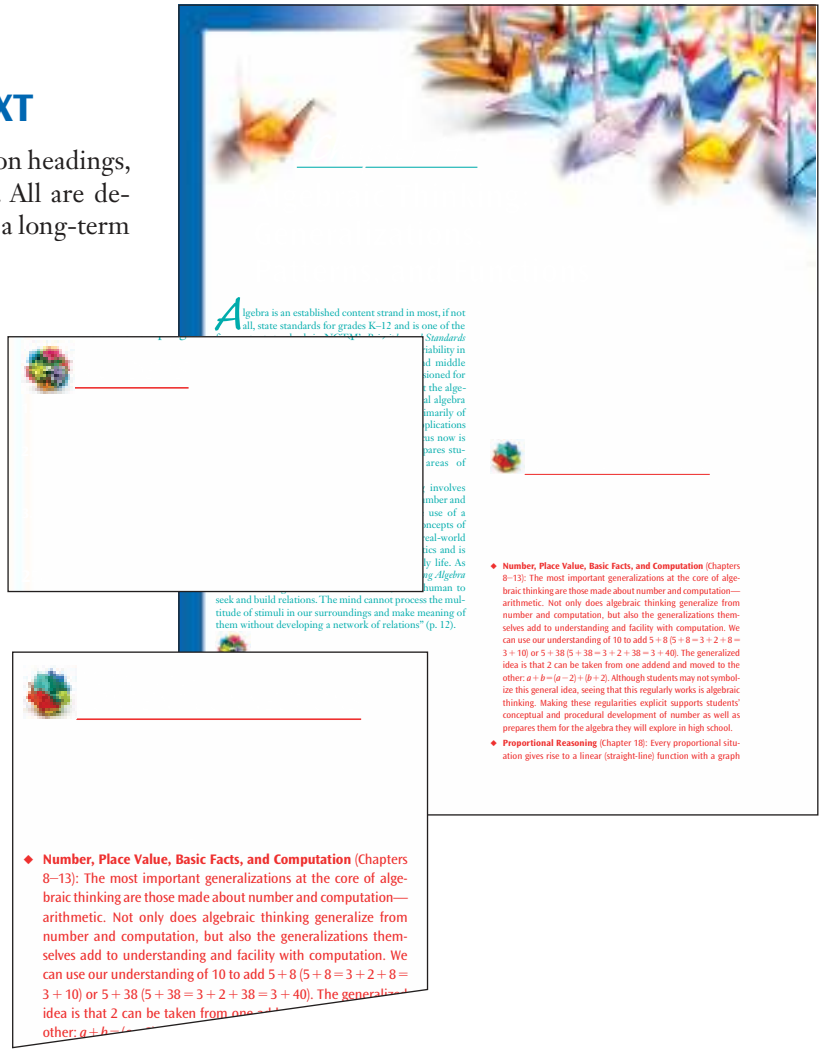
Each chapter of Section II focuses on one of the major content areas in pre-K–8 mathematics curriculum. It begins with identifying the big ideas for that content, and also provides guidance on how students best learn that content and many problem-based activities to engage students in understanding mathematics. Reflecting on the activities as you read can help you think about the mathematics from the perspective of the student. As often as possible, take out pencil and paper and try the problems so that you actively engage in *your learning* about *students learning* mathematics. In so doing, we hope this book will increase your own understanding of mathematics, the students you teach, and how to teach them well.

SOME SPECIAL FEATURES OF THIS TEXT

By flipping through the book, you will notice many section headings, a large number of figures, and various special features. All are designed to make the book more useful as a textbook and as a long-term resource. Here are a few things to look for.

Big Ideas ▶

Much of the research and literature espousing a student-centered approach suggests that teachers plan their instruction around "big ideas" rather than isolated skills or concepts. At the beginning of each chapter in Section II, you will find a list of the key mathematical ideas associated with the chapter. Teachers find these lists helpful for quickly getting a picture of the mathematics they are teaching.



◆ **Number, Place Value, Basic Facts, and Computation** (Chapters 8–13): The most important generalizations at the core of algebraic thinking are those made about number and computation—arithmetic. Not only does algebraic thinking generalize from number and computation, but also the generalizations themselves add to understanding and facility with computation. We can use our understanding of 10 to add 5 + 8 (5 + 8 = 3 + 2 + 8 = 3 + 10) or 5 + 38 (5 + 38 = 3 + 2 + 38 = 3 + 40). The generalized idea is that 2 can be taken from one addend and moved to the other: $a + b = (a - 2) + (b + 2)$. Although students may not symbolize this general idea, seeing that this regularly works is algebraic thinking. Making these regularities explicit supports students' conceptual and procedural development of number as well as prepares them for the algebra they will explore in high school.

◆ **Proportional Reasoning** (Chapter 18): Every proportional situation gives rise to a linear (straight-line) function with a graph

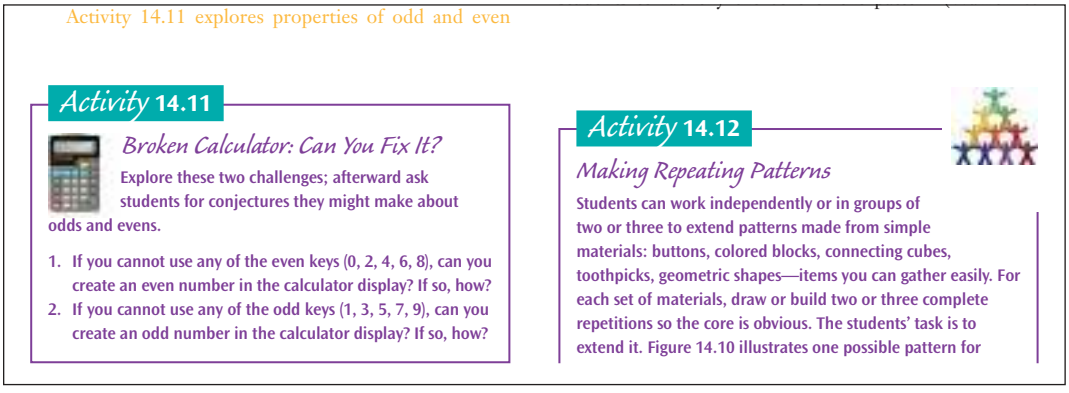
Mathematics Content Connections ▶

Following the Big Ideas lists are brief descriptions of other content areas in mathematics that are related to the content of the current chapter. These lists are offered to help you be more aware of the potential interaction of content as you plan lessons, diagnose students' difficulties, and learn more yourself about the mathematics you are teaching.

◆ **Number, Place Value, Basic Facts, and Computation** (Chapters 8–13): The most important generalizations at the core of algebraic thinking are those made about number and computation—arithmetic. Not only does algebraic thinking generalize from number and computation, but also the generalizations themselves add to understanding and facility with computation. We can use our understanding of 10 to add 5 + 8 (5 + 8 = 3 + 2 + 8 = 3 + 10) or 5 + 38 (5 + 38 = 3 + 2 + 38 = 3 + 40). The generalized idea is that 2 can be taken from one addend and moved to the other:

Activities ▶

The numerous activities found in every chapter of Section II have always been rated by readers as one of the most valuable parts of the book. Some activity ideas are described directly in the text and in the illustrations. Others are presented in the numbered Activity boxes. Every activity is a problem-based task (as described in Chapter 3) and is designed to engage students in doing mathematics. New adaptations and accommodations for English language learners and students with disabilities are included in many activities.



Activity 14.11 explores properties of odd and even

Activity 14.11

Broken Calculator: Can You Fix It?

Explore these two challenges; afterward ask students for conjectures they might make about odds and evens.

1. If you cannot use any of the even keys (0, 2, 4, 6, 8), can you create an even number in the calculator display? If so, how?
2. If you cannot use any of the odd keys (1, 3, 5, 7, 9), can you create an odd number in the calculator display? If so, how?

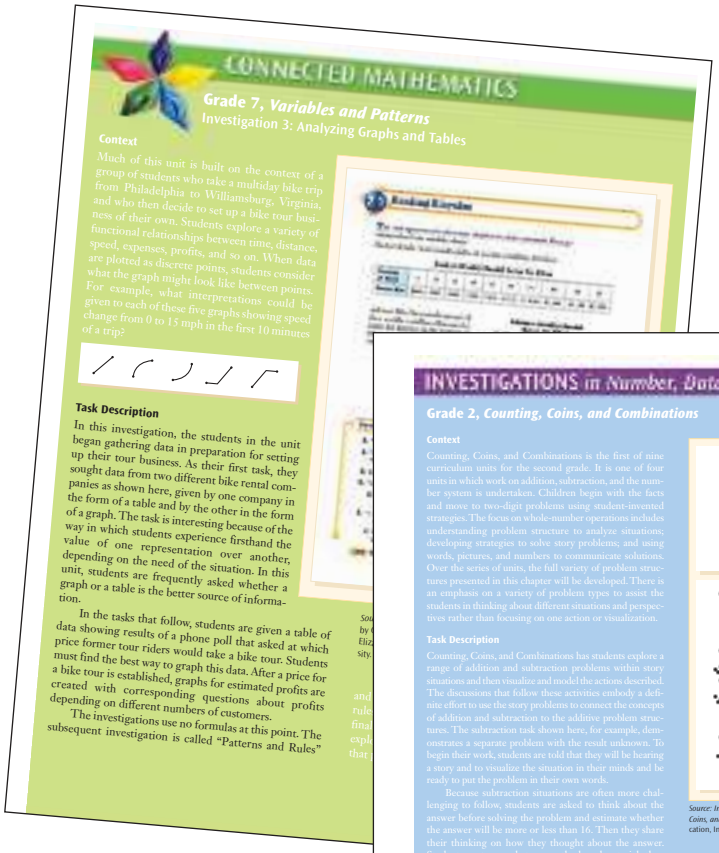
Activity 14.12

Making Repeating Patterns

Students can work independently or in groups of two or three to extend patterns made from simple materials: buttons, colored blocks, connecting cubes, toothpicks, geometric shapes—items you can gather easily. For each set of materials, draw or build two or three complete repetitions so the core is obvious. The students' task is to extend it. Figure 14.10 illustrates one possible pattern for

Investigations in Number, Data, and Space and *Connected Mathematics* ▶

In Section II, four chapters include features that describe an activity from the standards-based curriculum *Investigations in Number, Data, and Space* (an elementary curriculum) or *Connected Mathematics Project (CMP II)* (a middle school curriculum). These features include a description of an activity in the program as well as the context of the unit in which it is found. The main purpose of this feature is to acquaint you with these materials and to demonstrate how the spirit of the NCTM *Standards* and the constructivist theory espoused in this book have been translated into existing commercial curricula.


CONNECTED MATHEMATICS

Grade 7, *Variables and Patterns*
Investigation 3: Analyzing Graphs and Tables

Context

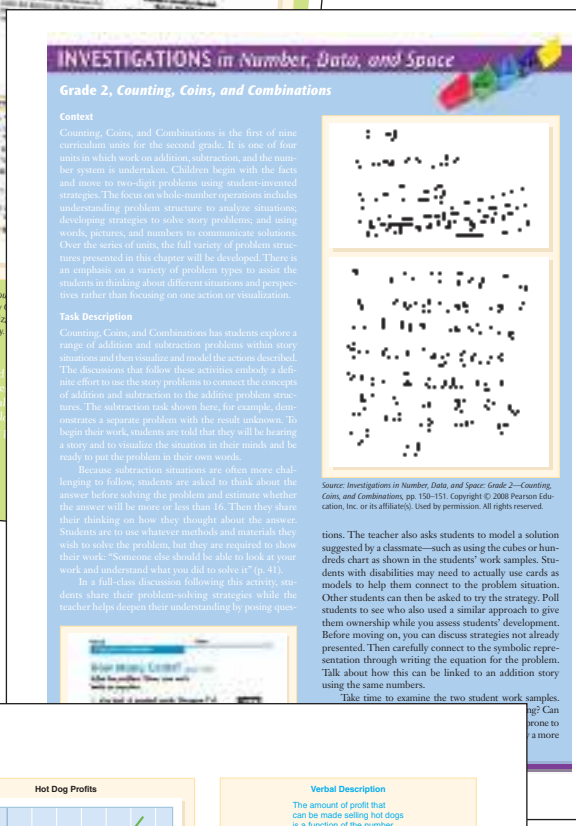
Much of this unit is built on the context of a group of students who take a multiday bike trip from Philadelphia to Williamsburg, Virginia, and who then decide to set up a bike tour business of their own. Students explore a variety of functional relationships between time, distance, speed, expenses, profits, and so on. When data are plotted as discrete points, students consider what the graph might look like between points. For example, what interpretations could be given to each of these five graphs showing speed change from 0 to 15 mph in the first 10 minutes of a trip?

Task Description

In this investigation, the students in the unit began gathering data in preparation for setting up their tour business. As their first task, they sought data from two different bike rental companies as shown here, given by one company in the form of a table and by the other in the form of a graph. The task is interesting because of the way in which students experience firsthand the value of one representation over another, depending on the need of the situation. In this unit, students are frequently asked whether a graph or a table is the better source of information.

In the tasks that follow, students are given a table of data showing results of a phone poll that asked at which price former tour riders would take a bike tour. Students must find the best way to graph this data. After a price for a bike tour is established, graphs for estimated profits are created with corresponding questions about profits depending on different numbers of customers.

The investigations use no formulas at this point. The subsequent investigation is called "Patterns and Rules"


INVESTIGATIONS in Number, Data, and Space

Grade 2, *Counting, Coins, and Combinations*

Context

Counting, Coins, and Combinations is the first of nine curriculum units for the second grade. It is one of four units in which work on addition, subtraction, and the number system is undertaken. Children begin with the facts and move to two-digit problems using student-invented strategies. The focus on whole-number operations includes understanding problem structure to analyze situations; developing strategies to solve story problems; and using words, pictures, and numbers to communicate solutions. Over the series of units, the full variety of problem structures presented in this chapter will be developed. There is an emphasis on a variety of problem types to assist the students in thinking about different situations and perspectives rather than focusing on one action or visualization.

Task Description

Counting, Coins, and Combinations has students explore a range of addition and subtraction problems within story situations and then visualize and model the actions described. The discussions that follow these activities embody a definite effort to use the story problems to connect the concepts of addition and subtraction to the additive problem structures. The subtraction task shown here, for example, demonstrates a separate problem with the result unknown. To begin their work, students are told that they will be hearing a story and to visualize the situation in their minds and be ready to put the problem in their own words.

Because subtraction situations are often more challenging to follow, students are asked to think about the answer before solving the problem and estimate whether the answer will be more or less than 16. Then they share their thinking on how they thought about the answer. Students are to use whatever methods and materials they wish to solve the problem, but they are required to show their work: "Someone else should be able to look at your work and understand what you did to solve it" (p. 41).

In a full-class discussion following this activity, students share their problem-solving strategies while the teacher helps deepen their understanding by posing ques-

Source: Investigations in Number, Data, and Space: Grade 2—Counting, Coins, and Combinations, pp. 150–151. Copyright © 2008 Pearson Education, Inc. or its affiliate(s). Used by permission. All rights reserved.

tions. The teacher also asks students to model a solution suggested by a classmate—such as using the cubes or hundreds chart as shown in the students' work samples. Students with disabilities may need to actually use cards as models to help them connect to the problem situation. Other students can then be asked to try the strategy. Poll students to see who also used a similar approach to give them ownership while you assess students' development. Before moving on, you can discuss strategies not already presented. Then carefully connect to the symbolic representation through writing the equation for the problem. Talk about how this can be linked to an addition story using the same numbers.

Take time to examine the two student work samples.

Technology Ideas ▶

Infusing technological tools is important in learning mathematics, as you will learn in Chapter 7. We have infused technology ideas throughout Section II. An icon is used to identify those places within the text or activity where a technology idea or resource is discussed. Descriptions include open-source (free) software, applets, and other Web-based resources, as well as calculator ideas.

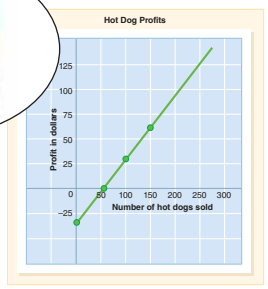


FIGURE 14.25 A graph showing profit as a function of hot dogs sold.

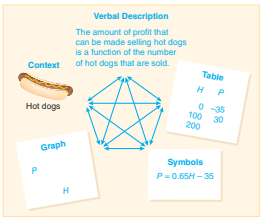


FIGURE 14.26 Five different representations of a function. For any given function, students should see that all these representations are connected and illustrate the same relationship.

The seventh-grade *Connected Mathematics Project* (CMP II) has an entire unit titled "Variables and Patterns," in which students explore and use different representations of functions in real contexts. The lesson shown here focuses on tables and graphs.

Algebraic Thinking Across the Curriculum

One reason the phrase "algebraic thinking" is used instead of "algebra" is that the practice of looking for patterns, regularity, and generalizations goes beyond curriculum topics that are usually categorized as algebra topics. You have already experienced some of this integration—looking at geometric growing patterns and working with perimeter and area. In fact, in *Curriculum Focal Points* (NCTM, 2006), many of the focal points that include algebra connect it to other content areas. In the sections that follow, the emphasis of the content moves to the other content areas, with algebraic thinking used as a tool for discovery. This brief discussion will be developed more fully in later chapters.

Measurement and Algebra. Soares, Blanton, and Kaput (2006) describe how to "algebraify" the elementary curriculum. One measurement example they give uses the children's book *Spaghetti and Meatballs for All* (by Burns and Tilley), looking at the increasing number of chairs needed given the growing number of tables.

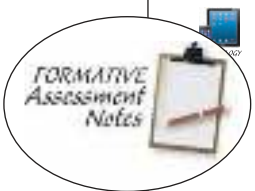
Geometric formulas relate various dimensions, areas, and volumes of shapes. Each of these formulas involves at least one functional relationship. Consider any familiar formula for measuring a geometric shape. For example, the circumference of a circle is $c = 2\pi r$. The radius is the independent variable, and circumference is the dependent

FORMATIVE Assessment Notes

The hot dog problem is a good **performance assessment.** A good question (which can be adapted to any task) is, "Can you show me where in the table, the graph, and the equation you can find the profit for selling 225 hot dogs?" ■

Formative Assessment Notes ▶

Assessment should be an integral part of instruction. Similarly, it makes sense to think about what to be listening for (assessing) as you read about different areas of content development. Throughout the content chapters, you will see Formative Assessment Note icons indicating a short description of ways to assess the topic in that section. Reading these assessment notes as you read the text can also help you understand how best to help your students.



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